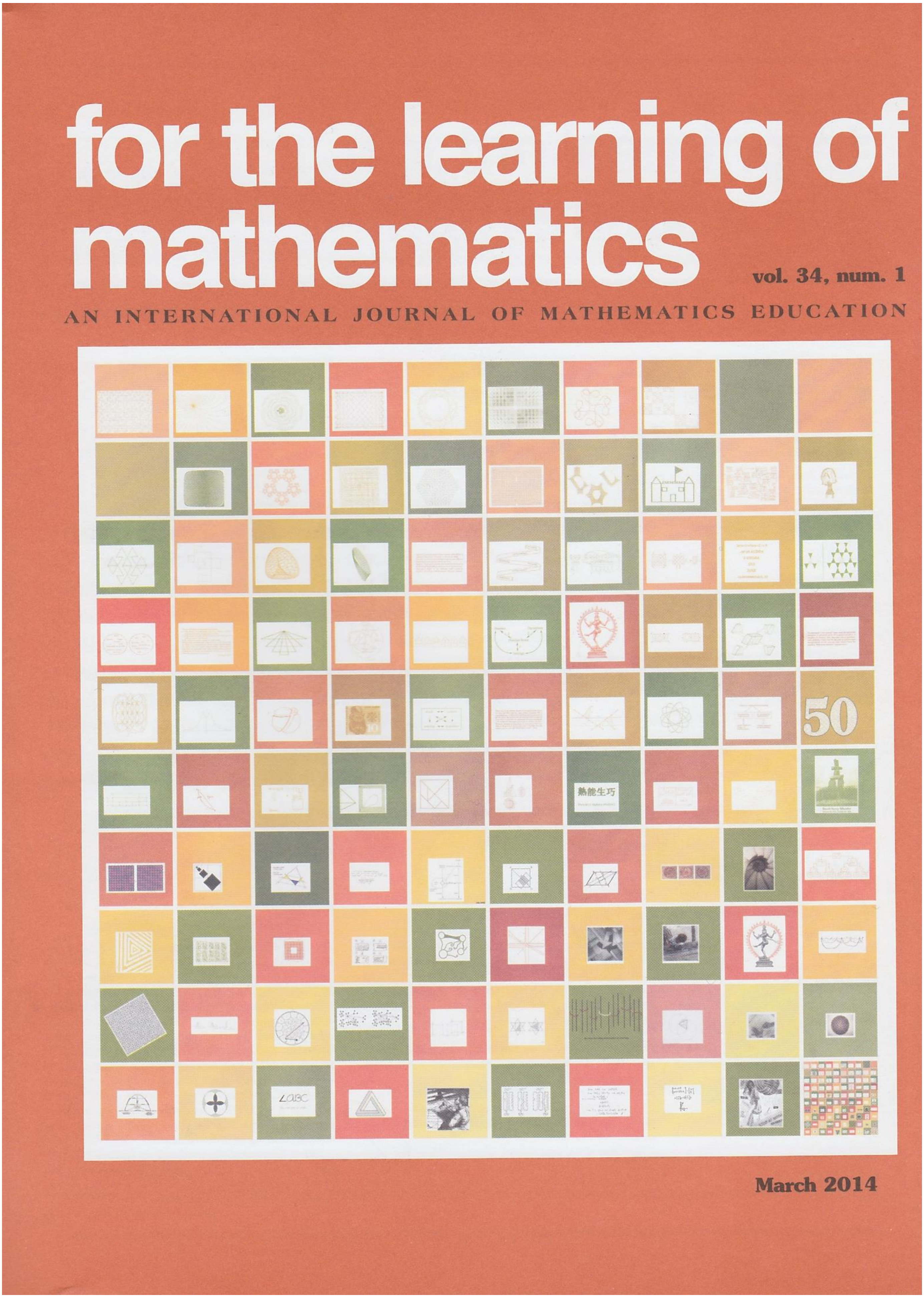
Goossen Karssenberg - Learning geometry by designing Persian mosaics



FLM PUBLISHING ASSOCIATION
FACULTY OF EDUCATION
UNIVERSITY OF NEW BRUNSWICK
P.O. BOX 4400
FREDERICTON, NEW BRUNSWICK
CANADA E3B 5A3

for the learning of mathematics

AN INTERNATIONAL JOURNAL OF MATHEMATICS EDUCATION

Volume 34, number 1

The image on the cover was prepared by Yasmine Abtahi. Lesley Lee kindly supplied back issues missing from the Editor's collection.

ISSN 0228-0671

Printed and bound in Canada.

EDITORIAL

RICHARD BARWELL

There are many possible representations of one hundred, 34(1) being one for which the connection may not immediately be obvious. Nevertheless, it is an anniversary of sorts. The hundredth issue of a journal that began with David Wheeler in 1980 and that has continued, with one or two minor hiccups, to produce, thus far, thirty-three volumes, each of three issues, containing for the most part thoughtful, often provocative writing for the learning of mathematics.

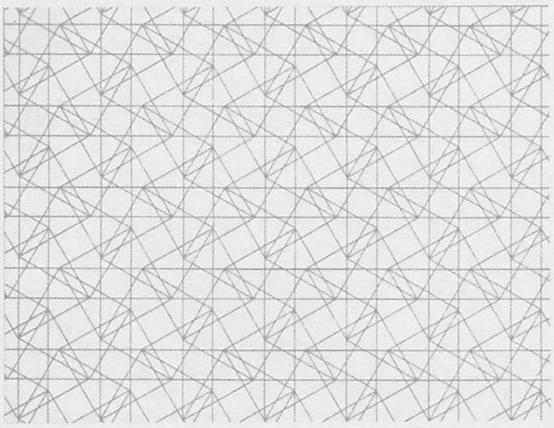
An anniversary can be a time for sober reflection on past achievements, for taking stock, perhaps a bit too much concerned with the past. For this issue, I wanted some attention to be given to the future. I contacted members of the advisory board and invited them to offer writing "for the learning of mathematics for the future". I did not impose any strict interpretation of this topic. For myself, I was influenced by Ubi d'Ambrosio's remark that as mathematics educators, we have a responsibility "for the future". What is the nature of this responsibility? How should we discharge it? What might be the future of learning mathematics? This hundredth issue contains some of the responses I received, including one from Ubi, as well as regular submissions, some of which seemed to me to fit the theme. Some additional responses will appear in the next issue and I should be happy to receive additional writing that addresses this topic, or that responds to the contributions in this issue.

Having brought FLM into the world, David Wheeler edited the first fifty issues, thus establishing its existence, nature and values. In his fiftieth and final issue, Wheeler wrote that it was time for the journal to make its way in the world: "The child was no longer an infant and was undoubtedly ready to assume some independence." It has taken four of us to edit the next fifty issues: David Pimm, Laurinda Brown, Brent Davis and me. We have guided FLM through its adolescence, perhaps, and it has now attained a degree of maturity; still aware of its parentage, but able to look after itself. As editor, I can only work with what is submitted, and yet the conversations and themes emerge, often taking FLM in directions that I have not anticipated. FLM, then, is no longer simply "David Wheeler's journal". It is its own entity.

This particular issue contains writing by a contributor to the very first issue (which you can find on the FLM website) and two who contributed to the fiftieth. Various other connections to the history of FLM appear throughout this issue, some more obvious than others. It also includes several authors contributing for the first time and some provocative and original thinking.

The start of Volume 34 marks the departure of Steve Lerman as one of the two associate editors. I am grateful for his participation in the editorial team for the past three years. Nathalie Sinclair continues in the role and is joined by David Reid.

And so, as we start on the next fifty issues, what will the future bring? What will the future bring, for example, for Lila, Mellony Graven's daughter, who features in her contribution, written with Steve Lerman? What will the future be for the learning of her mathematics, in the world she will inhabit? And what will the future be for our field? What will be the focus of our research? And how will we communicate our research and interact? Some responses, necessarily provisional, can be found in the contributions to this issue, as well as, of course, in *For the Learning of Mathematics* in the future.



For the Learning of Mathematics **34**, 1 (March, 2014)
FLM Publishing Association, Fredericton, New Brunswick, Canada

LEARNING GEOMETRY BY DESIGNING PERSIAN MOSAICS

GOOSSEN KARSSENBERG

The mosaic in Figure 1 shows rotational and mirror symmetries and mathematically interesting shapes like kites, regular 4- and 8-pointed stars and semi-regular octagons. Besides this, it has a specific sub-grid (Figure 1b). Mosaics like these can be found across much of the Islamic world, from Morocco to India. They are rarely used in mathematics lessons. In the present Dutch educational context, this situation is pitiful: due to immigration from Morocco and Turkey, during the last four decades the number of students with an Islamic background has increased significantly. Using examples which stem from these students' original cultures could stimulate their interest in mathematics. For students from any cultural background, the use of Islamic geometric designs in their mathematics class can improve intercultural awareness as well as an awareness of the links between mathematics, art and history.

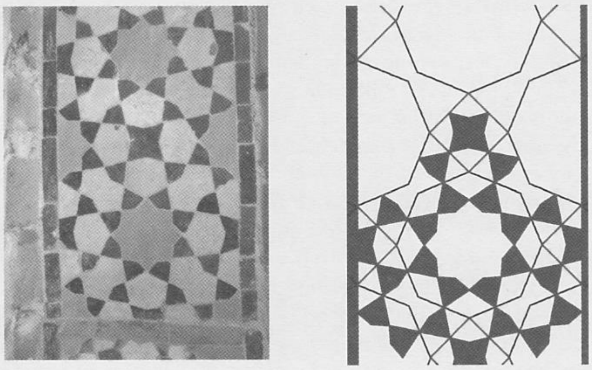


Figure 1. (a) Mosaic from Isfahan, Iran, and (b) a partial copy showing a sub-grid.

As Vithal and Skovsmose (1997) point out, however, there is a danger in taking subjects and methods out of their context and transplanting them into the context of "Western" mathematics education. Nevertheless, the use of non-Western mathematical sources can inspire teachers and students. As Zaslavsky (1999) writes, by doing so:

- "Students become aware of the role of mathematics in all societies. They realize that mathematical practices arose out of people's real needs and interests.
- "Students learn to appreciate the contributions of cultures different from their own, and to take pride in their own heritage.
- "By linking the study of mathematics with history, language, fine arts, and other subjects, all the disciplines take on more meaning." (p. 318)

My position is also inspired by the work of Katz (1998) and Gerdes (1998, 2010): there is more to mathematics than the Greek geometrical tradition, with its 17th and 18th century algebraic and analytical continuation (the triad of Euclid, Descartes and Euler). The work of medieval Islamic artists involves a rich application of geometry which students sense.

In this article, I report on how Dutch secondary school students actively investigated Islamic mosaics using mathematical means. This approach linked geometry teaching and learning with the history of geometry, and with the geometrical aspects of art and design. It also linked with the geographical and religious background of the mosaics. When the mosaics were designed, between the 8th and 17th centuries, the link with religion was obvious: Islamic artists were restricted in their choice of subjects. Besides holy texts and floral elements, only the use of abstract patterns was allowed and Islamic geometry with its focus on the circle and regular polygons provided rich grounds for their work.

As a whole, my approach, with its non-prototypical mathematical subject, student activities and assessment, can be seen as an example of "critical mathematical education" (Skovsmose, 1994). Through their investigations, the students showed pride in the culture of their ancestors, they acted as if they themselves were medieval designers and they learned geometry with pleasure. "Learning by acting" is a crucial aspect of this approach and is discussed more fully, below.

The place of the geometric design tradition in the history of mathematics

Since the very beginning of Islamic culture, religious buildings were decorated with geometric patterns. Hundreds of different types of decoration, often in combination with calligraphy and floral patterns, form an irreplaceable part of Islamic architecture. In addition to architecture, geometric patterns appear in manuscripts and carpets.

The intricacy of many designs presupposes a deep understanding of geometrical construction. One of the few primary sources about construction is a report about gatherings held in the 10th century (Özdural, 1996). Scholars with a background in mathematics and astronomy met with mosaic designers to discuss geometric construction issues. The report shows that the tradition of mosaic design stood on its own as a field of study, with, as its main goal, the designing of beautiful patterns. While the mathematicians were interested in giving proofs in the style of Euclid, the mosaic designers allowed approximate constructions.

The production process of the geometric designs can be divided into three stages. First, a creative process took place in which a specific pattern with certain constraints is designed. In this process various geometrical skills, such as, for example, construction techniques for regular pentagons, were applied (Hogendijk, 2012). Second, a drawing plus instructions was produced, from which the newly designed mosaic could be reproduced. In fact, the instructions barely show how the design process took place. Unfortunately, most of the time, the few sources that we have show drawings without written instructions. Most famous of these is the Topkapi scroll, an approximately 400-year old document containing dozens of drawings of geometrical patterns (Necipoğlu, 1995). Third, the pattern was constructed to scale by artisans, coloured, sometimes decorated further using calligraphy or floral elements, on paper, in stucco or using cut-tiling (as in Figure 1a).

There are hundreds of primary sources concerning the third stage but only five or so for the second stage. There are no sources at all concerning the first stage. This fact could be used to interest school students in working on the geometric designs. Provided that it is presented to the class in the right way, students can sense the feeling of reinventing mathematical design techniques (an idea also proposed by Gerdes, 1988). This was the starting point for the activities described in this article.

The Dutch setting

I worked in schools that prepare their students for university. The students of one class, average age 16 years, had little interest in mathematics and had opted for a career focusing on social, cultural or economic studies. Three students in the class focused even more on culture and society: their subject package included art, geography and history. The students followed a mathematics curriculum that mainly consisted of probability, calculus, statistics and basic algebra. They often worked alone or in pairs and most of their motivation originated from their focus on passing their exams.

Given the circumstances in this class, imagine the following scenario: the students are busy in small mixed groups designing Islamic-style mosaics, relating their work to the medieval design tradition and analysing their designs mathematically, after which they present their work to an audience from their school community. An activity like this, I reasoned, would motivate those with an Islamic background, as well as those interested in art or history. Moreover, while the students are working on a fundamental part of mathematics, as a side effect it would promote intercultural awareness.

With this idea in mind, I designed a series of lessons based on the geometric design process and tried it out in May 2010 in a pilot project at two schools. The pilot involved 12 students aged 15-16 years from an Islamic background in a school in Rotterdam, and 16 students aged 16-17 years from the island of Texel, a rural part of the Netherlands with a mainly indigenous Dutch population. A year later, an improved version of the course was taken by a total of 30 students from the same two schools. In 2012 and 2013 two more cycles with revisions ran on Texel (18 and 20 students) and in The Hague (50 students in 2013). In 2013 the course also ran in two classes in Zürich, Switzerland.

The didactical approach: learning by acting

The design principles behind the lesson strand are based on the conceptual framework of activity. Cole and Engeström (1993) proposed expanding the basic mediational triangle to model any activity system. This framework sheds light on the possible merits and tensions of the chosen teaching strategy. It explains why students get involved and take pride in their work.

The triangle is used here to define a didactical tool, that I call "learning by acting". The object of this activity is to prepare a "play". In the triangle, the word "play" is shown in inverted commas since often there is no formal performance: rather, the students play a game together without any spectators except for the teacher and the students themselves [1]. "Play" may, for example, be a so-called "management game": business is taught by having students set up fake businesses and trade with their classmates. Or, when learning probability theory, students may be asked to act as if they were members of an insurance company or gamblers. In physics and chemistry education, this tool is called "learning in an authentic practice", which is quite common nowadays. Figure 2 shows the mediational triangle for a student (the *subject*) who learns in the setting of a "play".

The main advantage of learning by acting is that the *outcome* is clear to both the student and the teacher: to perform a good "play" or to play the game successfully. This product is essential for student motivation. In many situations, the main object for a student in class is "classroom survival" (Jaworski, 2009). However, when the class as a whole, including the teacher, has one common goal, classroom survival is less of an issue and this goal prevails. In order to perform a good play, the student needs appropriate *tools*. In the example of the gamblers, tools could be dice and coins, as well as mathematical tools, such as information and exercises concerning probability.

Besides the common *rules* set by the teacher on how to behave in class, the teacher explains the rules of the game. For instance, in a historical play, the teacher divides roles among students, structures rehearsals, *etc*. Informal rules stem from group authority structures in class. The *division of labour* comprises, in line with Jaworski's reasoning (2009), formal and informal aspects: formal power rests with the teacher who gives tasks, assesses and stimulates by giving positive feedback or handing out prizes for the best performances. Informal power rests with the students who can choose how to react. The *community* can play an important role when they are invited to be spectators of the performance. This can motivate students to deliver a good product.

This teaching tool needs certain conditions to work effectively. Most important among these is that the students must be able to grasp the role they play: they must have basic knowledge of the topic. A second important condition is that the students should use material that fits the setting in which they act.

Learning mathematics by acting as mosaic designers

In this example, learning by acting is used in a weaker form: students act as if they were professional medieval artists, who produce and present a mosaic. The play invites them to study medieval art and its techniques. Since the students are focused on giving interesting presentations, it is highly stimulating to have them actually make a colourful mosaic. By working together and completing one mosaic per group, the amount of time needed per student is reduced.

The mediational triangle for a student who participates in the geometric mosaic lesson strand involves the following components.

Subject: The student.

Tools: Triangle, compass, theoretical and practical information about mosaic design, exercises, examples of Islamic mosaics, coloured paper for making tiles.

Objects: To gain skills in designing mosaics with traditional tools. To collectively design and produce a mosaic, including a mathematical analysis of the design made by the group members.

Extra rules: The teacher divides the class into groups; students parcel out the tasks and make arrangements with group members. The teacher announces how the posters and the presentation are assessed.

Communities: The group of designers with whom the mosaic is made and presented (2-4 students). Family, friends and fellow students as an audience for the presentation.

Division of labour: The teacher gives tasks to design a mosaic and prepare a presentation. Formal power rests with the teacher who assesses the presentations and posters. Power in practice rests with the students who can choose how to respond in accordance with group pressure.

Outcome: Presentation of the mosaic including the design process and an analysis of the design. This demonstrates the knowledge gained by students of geometrical construction, symmetry and history of ornamentation and geometry.

The analysis of the mosaic from a modern perspective would hinder the students taking up the role of a medieval Persian designer. A modern perspective is represented, for instance, by Abas and Salman (1995), who analyse mosaics using the 17 types of tessellating wallpaper patterns. However, it is almost certain that medieval designers were unfamiliar with this typology of tessellations. Also, this typology falls short when it comes to analysing and cataloguing Islamic mosaics. Therefore, in the next section I propose a way of analysing mosaic designs that relates more directly to the medieval tradition and enables the students to quickly become familiar with Islamic mosaics.

Mathematical tools

Before the students could act as if they were traditional mosaic designers they would need to become familiar with some geometrical methods and construction tools. Here is a short resume of the preparation program they followed. It is not based on any pre-prepared program. Only indirect sources are available: the mosaics themselves and some manuscripts.

The lesson strand, about 12 lessons in total, begins with a historical introduction and some simple examples of

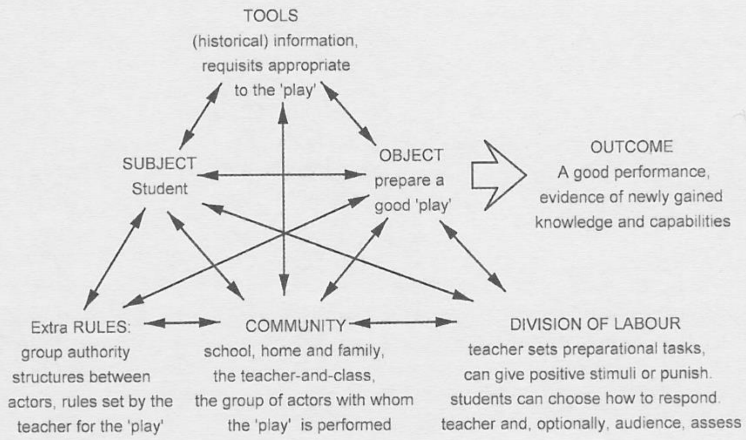


Figure 2. Expanded mediational triangle (Cole and Engeström, 1993) for a student in a learning by acting setting.

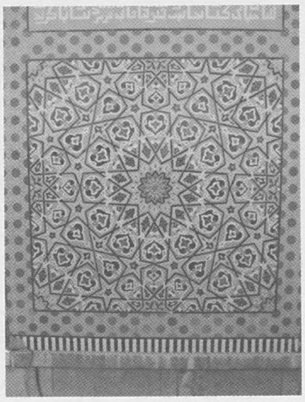


Figure 3. Recent elaboration of the mosaic analysed in Figure 4 (Photo: Jan Hogendijk, Yazd, Iran).

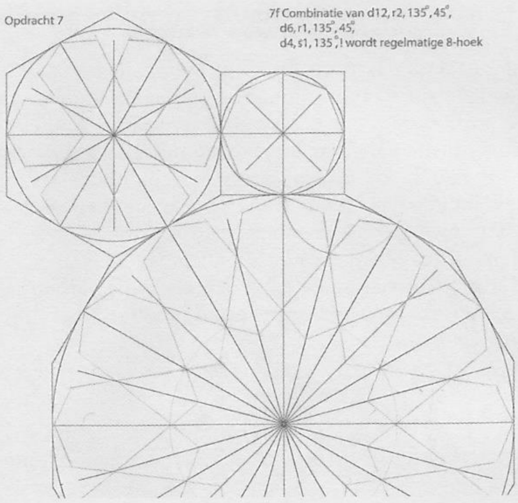


Figure 5. Students practice drawing techniques for rosettes, on a sheet with given polygons.

medieval mosaics which the student can draw using folding techniques. By working on these activities for three lessons, the students become acquainted with the topic. Subsequently, the students practice how Islamic mosaics can be analysed using seven concepts. These concepts (referred to in roman numbers) are explained here with the help of a quite common mosaic. Figure 3 shows a modern example of this mosaic from Yazd (Iran); Figure 4 shows a diagram with the analysis of the same mosaic. The diagram shows part of the mosaic and (to the left) its symmetries.

The concept of global symmetry is new to most students: the fact that there are lines of mirror symmetry (*I*, dotted lines) and points of rotation symmetry (*II*, points with *G*-numbers, where *G* stands for "global symmetry") outside the centre of the mosaic (since the mosaic can be extended infinitely) is not immediately clear to all and must be practiced.

Next, the new concept of *local* symmetry (*III*, dotted circles with *L*-numbers) is introduced. This concept describes what can be found in many Islamic mosaics, especially when it comes to local *rotational* symmetry: local areas with rotational symmetry are identified by circles and a number. For instance: *L9* means that the part of the mosaic inside the circle, when turned one-ninth of a full turn, overlaps its original exactly. In practice, only local symmetries of order three or higher are shown. Circles with local symmetry can overlap and their midpoints often coincide with centres of global rotational symmetry of lower order.

Although it cannot be proved that designers made use of these or similar concepts, I found them a highly appropriate and effective way to gain familiarity with the essence of Islamic mosaics. Furthermore, local symmetry is a promising concept with which to categorize Islamic tessellations.

Almost all Islamic mosaics show translational symmetry

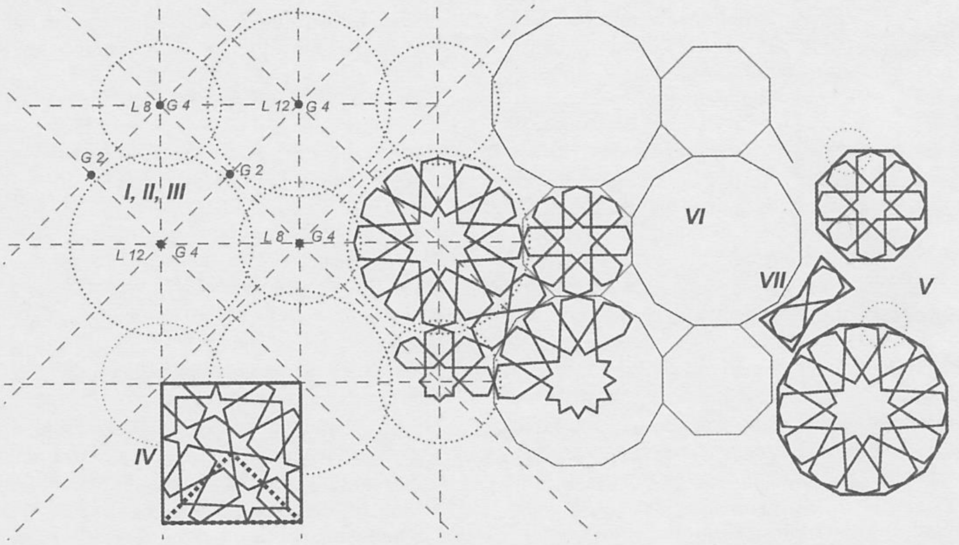


Figure 4. Seven aspects of many Islamic mosaics shown in one example.

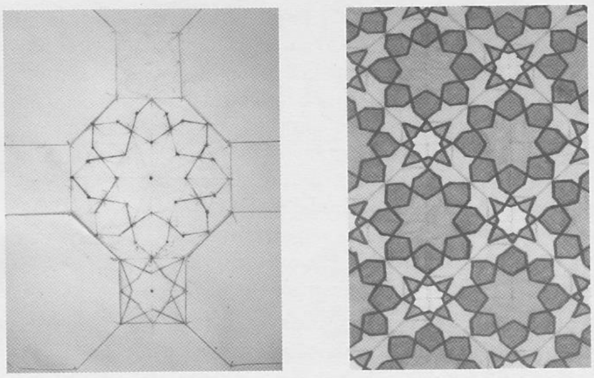


Figure 6. (a) Basic pattern and (b) the end result (detail).

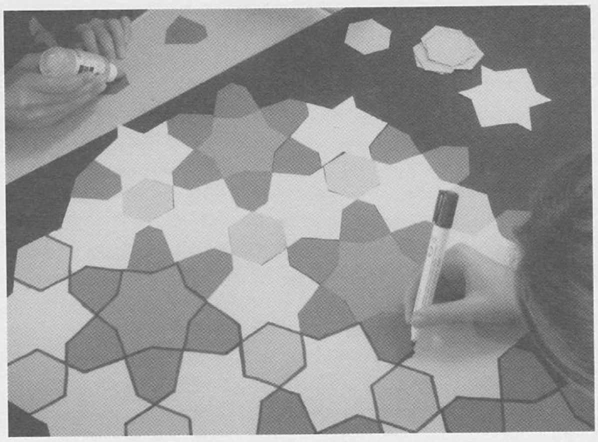


Figure 7. Students use coloured paper tiles.

in two directions, meaning that a rectangular part can be identified as a repeat unit or cell (*IV*). Students learn to identify these in mosaics and are also challenged to find a so-called smallest cell. This is the smallest area possible with which, together with its reflection if necessary, the whole mosaic can be constructed (in the student work shown in Figure 8, it is the triangle in the upper right corner).

A practical way of defining and drawing regular stars is introduced. Next, students learn how to define and draw a broadly used class of rosettes (*V*, two types in Figure 4, student work in Figure 5). The orientation of the rosettes or stars in regular polygons, with vertices on the midpoints of the sides of the polygons, is important for its direct application in a specific mosaic design process, in which basic patterns are used (*VI*, Figure 4; student work in Figure 6a).

A basic pattern is, in my definition, a regular covering of the plane with a finite number of different (approximately) equilateral polygons. Nowadays, these polygons are also called *girih*-tiles (from the Persian word *gīreh*, knot). In many Persian tessellations, there is a basic pattern underlying the actual mosaic, often with at least one regular polygon in which a rosette or a star is placed (Cromwell 2009). Figure 1 shows another example, in which four- and eight-pointed stars were placed in the regular polygons. In the course, students learn how to find basic patterns in original mosaics.

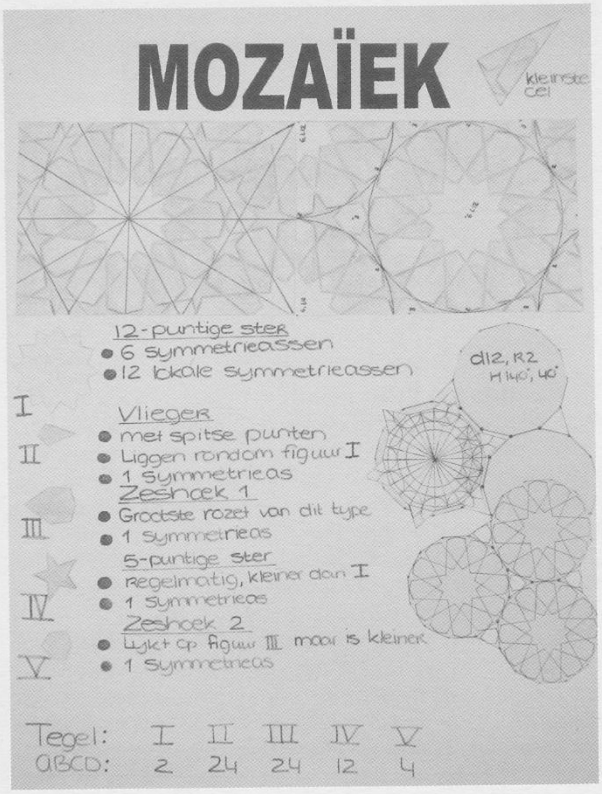


Figure 8. Poster with analysis.

When using basic patterns to design mosaics in this style, some rules have to be obeyed rather strictly (*VII*). One of these is that lines should not "curve" more than, say, 10 degrees at their crossing points. There are also some looser rules on how to fill out non-regular polygons of the basic pattern relating to the goal of finding artistic balance in the whole design. By practicing with existing mosaics, students become familiar with the way this technique was used centuries ago and can apply it when designing new mosaics for themselves.

The students start acting as medieval Persian mosaic designers

After five lessons of practicing the above theory, both by analysing old mosaics and by drawing cells of simple mosaics and rosettes themselves, students proceeded to make first drafts of their own new mosaic designs with the use of a basic pattern. Subsequently, in small groups they chose one of their designs and finished the lesson strand in line with the learning by acting approach.

Figures 6-8 give an impression of the students' output [2]. All groups produced a mosaic and analysed it on a poster like the one by three students from Texel, shown in Figure 8. Short descriptions of the five different tiles are given and also their relative frequencies of appearance in the mosaic (at the bottom of the poster). The only mistake that I found was that the circles showing areas of local 12-fold rotation symmetry should have been slightly bigger.

The analysis (as in Figure 8) and the presentation for peers and relations as a *grande finale* were an essential part of the lesson strand. Students showed good understanding of what they had learnt about analysing mosaics. They showed local and global symmetries in their designs and identified stars and rosettes including accurate descriptions of the types. Sometimes certain claims were disputed by the audience, which led to interesting debates, for instance about whether the colouring is to be taken into account when looking for symmetries. Reflection about the final mosaics, making connections with the theory, and phrasing the conclusions contributed for the students to the internalization of their newly gained knowledge and skills.

Reflections on the results

Reflecting on the four lesson cycles, I find that the students grew in mathematical experience. They learnt to globally and structurally analyse complex geometrical figures, and applied their knowledge in actively designing and producing mosaics themselves. Furthermore, they communicated with other group members and with an audience in the final presentation. The whole arrangement worked well, as all connections in the mediation triangle were strong and effective.

The majority of the students were positive about the level of difficulty of the lesson strand; interviews showed that 8% found it "very difficult", while the vast majority felt happy with the results. The responses of the Muslim students did not differ significantly from those given by the others. When asked how much they learnt from the course as compared to their "normal" mathematics lessons, on mathematical knowledge the average score was "slightly less" and on general knowledge it was "more". This outcome might be due to the fact that the students did not consider the process of creating a geometric mosaic a mathematical activity, which is not surprising, since it is not in their books.

The motivation of all students was high. For those with an Islamic background this may have to do with their recognition of the topic in their daily life. For instance, one student said, "Sir, I've seen these kinds of mosaics on a mosque in Istanbul during our holiday last summer!" In presentations, students mentioned inspiration from their homes or even the help they received from a grandfather to decorate their mosaic with calligraphy. Both the teachers involved and the audiences of the presentations were enthusiastic about the project. The fact that the teachers plan to continue using the material shows that the energy put into getting familiar with the topic was rewarding.

Many students welcomed the change of lesson style and topic saying, "Wow! I never realized that this is maths as well!" and, "I like this a lot more than doing all these boring exercises alone!" Some of them were utterly surprised about the fact that one can find so much mathematics in "these simple mosaics". A small minority struggled in finding symmetries and drawing rosettes. Since they had the opportunity to choose relatively simple basic patterns for their own design (as in Figure 6a) and received help from their fellow students, their results were still quite good and these students also took pride in presenting their work in front of an audience.

Unfortunately, during the presentations, students did not discuss whether they gained a deeper understanding of how the traditional designers worked centuries ago. Although their output showed that the students did get a great acquaintance with the topic, they did not show awareness of this fact explicitly. This problem may be solved in the future by stressing this question more during class and requiring attention to links with history in the presentations.

There were some critical remarks from other teachers who were informed about the lesson strand. These can be best summarized with the following quotations:

- "Their goal is to make a beautiful mosaic. But what about gaining mathematical knowledge?"
- "It is too time consuming letting them finish a complete mosaic. This should be done during art, not mathematics classes."

My position is that the mathematical content is indeed unlike the mathematics that students learnt (or did not learn) before, but the mosaic activities deal with symmetry, construction and structuring information and are, as such, good examples of applied mathematics. Additionally, this content is highly appropriate to the target group (students who will not choose a technical or scientific program at university, some of whom are particularly interested in art). I agree with Eglash (1999), who proposes to speak of mathematical activity within art as soon as there is an "intentional application of explicit rules" (p. 49). Furthermore, Bartolini Bussi (2000, p. 348) suggests that the tactile element in working with instruments in mathematics education can enhance student learning. The same holds for the design and production of a colourful mosaic. This process is rewarding and confronts the students with the relation between mathematics and the fine arts. The students develop a positive motivation for being in class and also increase their awareness of links between mathematics, art and history. The effect on learning of the presentation is also strong: students have to analyse and verbalize their methods.

These student activities are quite different from what "normally" takes place during mathematics class. Both the students and their teacher need to find their new roles in the new setting where the student objective is to deliver a good presentation with group members rather than "to survive the classroom" or to pass a test. The arrangement in which both the object and the outcome differ a lot from what the students and teacher were used to, including the new roles for the community (see the mediational triangle in Figure 2) creates opportunities for new ways of learning, and new motivations for learning.

Conclusion

The use of the tradition of geometric design in the way described in this article is promising: both the students who were not familiar with Islamic culture beforehand and those who were, showed good motivation and created interesting patterns. The teachers and the students were positive about the set-up of the lesson strand. The goal of creating beautiful geometric, Persian-style mosaics in small groups and presenting them, including the accompanying mathematical analyses, stimulated the students to get involved. It cannot be proven that the goal to find out more about the way the traditional designers could have worked by making designs themselves was reached by the students. But, on the whole, the students did learn a lot about symmetry, construction techniques and ways of defining rosettes in a short time.

The presentation of the didactical tool "learning by acting" in a mediational triangle identifies ways to reduce tensions between the items in the triangle, thus enhancing the design of the lesson strand. The use of role-play allocates the position of the student who tries to act as a medieval mosaic designer and uses appropriate tools. One is tempted to call these tools ethnomathematical but I would rather see them in a historical perspective, and call them contemporary, in the sense of "belonging to these early times". This strategy deserves attention in mathematics education: other topics can be dealt with in a learning by acting setting, and they can be combined with matching episodes from the history of mathematics or with contemporary practical mathematical problems.

Besides learning about geometry, symmetry and mathematical approaches to problems, the students learnt about the cultural and historical background of geometry and its links with architecture in Islamic culture. This work supports Zaslavsky's summary of how non-Western mathematical sources can inspire teachers and students, as quoted at the start of this article. By focusing on the mathematical content, the tradition of Islamic geometric decoration can be used effectively in mathematics classes, irrespective of the students' cultural backgrounds. As an additional advantage, it improves intercultural awareness.

I conclude with the issue of ethnomathematics and mathematics education. Our lesson strand was first and for all an attempt to give sound mathematics education. The inspiration came from far away, in subject, method, geography and in time. These distances did not hinder; rather, they were a stimulus. With the choice to let students work on the topic of medieval Islamic art, I followed the approach advocated by Gerdes, Zaslavsky and Katz: non-Western cultures have produced mathematics and everyone can profit from its inspiration. In our classes, at least, it worked out well.

Notes

[1] "Play" may also encompass actual drama on stage, as shown, for example, by Ponza (2000), who describes algebra lessons in which the life and death of Galois are staged and gives further references.

[2] More pictures can be found at www.goossenkarssenberg.nl.

Acknowledgements

This project was financed by the Netherlands Organisation for Scientific Research (NWO), "Teacher in research" (LIO-) program nr. 636.000.043, title: "Middeleeuwse islamitische mozaïeken als lesmateriaal". It was carried out at Utrecht University (Freudenthal Institute for Science and Mathematics Education and Mathematical Institute). I wish to thank Marjanne de Nijs, Marga Loos and Peter Teubel for their commitment, Tom Goris for guiding me in Iran, all people of the House of Mathematics, Isfahan, for their hospitality, my supervisors Jan P. Hogendijk and Jan van Maanen for their stimulating help and all four anonymous reviewers for their insightful critics.

References

Abas, S. J. & Salman, A. S. (1995) *Symmetries of Islamic Geometrical Patterns.* Singapore: World Scientific Publishing.

Bartolini Bussi, M. G. (2000) Ancient instruments in the modern classroom. In Fauvel, J. & Maanen, J. van (Eds.) *History in Mathematics Education,* pp. 343-350. Dordrecht, The Netherlands: Kluwer Academic Publishers.

Cole, M. & Engeström, Y. (1993) A cultural-historical approach to distributed cognition. In Salomon, G. (Ed.) *Distributed Cognition: Psychological and Educational Considerations,* pp. 1-46. New York, NY: Cambridge University Press.

Cromwell, P. R. (2009) The search for quasi-periodicity in Islamic 5-fold ornament. *Mathematical Intelligencer* **31**(1), 36-56.

Eglash, R. (1999) *African Fractals: Modern Computing and Indigenous Design.* New Brunswick, NJ: Rutgers University Press.

Fauvel, J. & Maanen, J. van. (2000) *History in Mathematics Education: The ICMI Study.* Dordrecht, The Netherlands: Kluwer Academic Publishers.

Gerdes, P. (1998) On culture and mathematics teacher education. *Journal of Mathematics Teacher Education* **1**(1), 33-53.

Gerdes, P. (2010) Exploration of technologies, emerging from African cultural practices, in mathematics (teacher) education. *ZDM* **42**(1), 11-17.

Hogendijk, J. P. (2012) Mathematics and geometric ornamentation in the medieval Islamic world. *The European Mathematical Society Newsletter* **86**, 37-43.

Jaworski, B. & Potari, D. (2009) Bridging the macro-and micro-divide: using an activity theory model to capture sociocultural complexity and mathematics teaching and its development. *Educational Studies in Mathematics* **72**(2), 219-236.

Katz, V. (1998) *A History of Mathematics: An Introduction* (2nd edition). Reading, MA: Addison-Wesley.

Necipoğlu, G. (1995) *The Topkapi Scroll: Geometry and Ornament in Islamic Architecture.* Santa Monica, CA: Getty Center Publication.

Özdural, A. (1996) On interlocking similar or corresponding figures and ornamental patterns of cubic equations. *Muqarnas* **13**, 191-211.

Ponza, V. (2000) Mathematical dramatisation. In Fauvel, J. & Maanen, J. van. (Eds.) *History in Mathematics Education: The ICMI Study,* pp. 335-342. Dordrecht, The Netherlands: Kluwer Academic Publishers.

Skovsmose, O. (1994) Towards a critical mathematics education. *Education Studies in Mathematics* **27**(1), 35-57.

Vital, R. & Skovsmose, O. (1997) The end of innocence: a critique of 'ethnomathematics'. *Educational Studies in Mathematics* **34**, 131-158.

Zaslavsky, C. (1999) *Africa Counts, Number and Pattern in African Cultures* (3rd edition). Chicago, IL: Chicago Review Press.

Contributors

ANNA BACCAGLINI-FRANK
Dipartimento di Educazione e Scienze Umane
Università di Modena e Reggio Emilia
via Allegri, 9
42121 Reggio Emilia
Italy
(abaccaglinifrank@gmail.com)

MARIA G. BARTOLINI BUSSI
Dipartimento di Educazione e Scienze Umane
Università di Modena e Reggio Emilia
via Allegri, 9
42121 Reggio Emilia
Italy
(bartolini@unimore.it)

LAURINDA BROWN
Graduate School of Education
University of Bristol
35 Berkeley Square
Bristol BS8 1JA
UK
(laurinda.brown@bris.ac.uk)

EGAN J CHERNOFF
(@MatthewMaddux)
College of Education
University of Saskatchewan
28 Campus Drive
Saskatoon, SK, S7W 0A8
Canada
(egan.chernoff@usask.ca)

UBIRATAN D'AMBROSIO
Rua Peixoto Gomide, 1772 ap.83
01409-002 São Paulo, SP
Brazil
(ubi@usp.br)

BRENT DAVIS
Werklund School of Education
University of Calgary
2500 University Drive NW
Calgary, AB, T2N 1N4
Canada
(brent.davis@ucalgary.ca)

BARBARA J. DOUGHERTY
University of Missouri
122A Townsend Hall
Columbia, MO 65211
USA
(doughertyb@missouri.edu)

MELLONY GRAVEN
Education Department
Rhodes University
PO Box 94
Grahamstown, 6140
South Africa
(m.graven@ru.ac.za)

GOOSSEN KARSSENBERG
Den Burg
Texel
The Netherlands
(gkarssenberg@live.nl)

NORIO KOIKE
Nagano Prefectural Board of Education
692-2, Habashita, Minaminagano
Nagano City
Nagano Prefecture, 380-8570
Japan
(koikenorio@gmail.com)

KOTARO KOMATSU
Faculty of Education
Shinshu University
6-ro, Nishinagano
Nagano City
Nagano Prefecture, 380-8544
Japan
(kkomatsu@shinshu-u.ac.jp)

LESLEY LEE
417 St Joseph Ouest, #14
Outremont, QC, H2V 2P3
Canada
(lesley.lee@mac.com)

STEPHEN LERMAN
Department of Education
London South Bank University
103 Borough Road
LONDON SE1 0AA
UK
(lermans@lsbu.ac.uk)

JEAN-FRANÇOIS MAHEUX
Département de mathématiques
Université du Québec à Montréal
C.P. 8888, Succ. Centre-Ville
Montréal, QC, H3C 3P8
Canada
(maheux.jean-francois@uqam.ca)

DAVID PIMM
Faculty of Education
Simon Fraser University
Burnaby, BC, V5A 1S6
Canada
(djp3@sfu.ca)

ALESSANDRO RAMPLOUD
Dipartimento di Educazione e Scienze Umane
Università di Modena e Reggio Emilia
via Allegri, 9
42121 Reggio Emilia
Italy
(alessandro.ramploud@gmail.com)

DAVID A. REID
FB3
University of Bremen
Bibliothekstraße 1
28359 Bremen
Germany
(dreid@uni-bremen.de)

ARUTA SAKAMAKI
Nagano Junior High School
(Attached to the Faculty of Education of Shinshu University)
109, Minamibori
Nagano City
Nagano Prefecture, 381-0016
Japan
(aruta_sakamaki@ybb.ne.jp)

ANNA SFARD
The Faculty of Education
University of Haifa
Mount Carmel
Haifa 31905
Israel
(annasfar@math.msu.edu)

NATHALIE SINCLAIR
Faculty of Education
Simon Fraser University
8888 University Drive
Burnaby, BC, V5A 1S6
Canada
(nathsinc@sfu.ca)

BHARATH SRIRAMAN
Department of Mathematical Sciences
University of Montana
Missoula, MT 59812
USA
(sriramanb@mso.umt.edu)

PETER TAYLOR
Department of Mathematics and Statistics
Queen's University
Jeffery Hall 513
Kingston, ON, K7L 3N6
Canada
(peter.taylor@queensu.ca)

YOSUKE TSUJIYAMA
Faculty of International Studies
Keiai University
1-5-21, Anagawa, Inage
Chiba City
Chiba Prefecture, 263-8588
Japan
(y-tsujiyama@u-keiai.ac.jp)

LINDA VENENCIANO
College of Education
University of Hawai'i, Manoa
1776 University Avenue
Honolulu, HI 96822
USA
(Lhirashi@hawaii.edu)

FLM 34(1)